Multifidelity Conceptual Design and Optimization of Strut-Braced Wing Aircraft using Physics-Based Methods

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Given the need to reduce fuel burn and emissions from aircraft drastically, aircraft designers are moving away from conventional tube-and-wing aircraft configurations towards unconventional configurations in search of benefits in terms of fuel burn and emissions. However, conceptual design and optimization of these configurations is still a challenging problem owing to the inability of correlation-based methods to accurately predict the aerodynamics and structural weight of the aircraft. In this paper we present a multi-fidelity design framework that uses finite element-based structural sizing and weight estimation, vortex lattice and CFD-based aerodynamics and automated parametric geometric modeling. These physics based methods are combined with a conceptual design framework to allow realistic design and optimization of unconventional aircraft configurations. We then use this physics-based design framework to perform design and optimization studies on a strut-braced-wing aircraft configuration.

I. Introduction

The reduction of fuel burn and emissions has been a major focus of aviation research in both academia and industry for quite a few years.\textsuperscript{1} Different design methodologies and technological improvements have been studied with the aim of reducing aircraft fuel burn. The effect of making changes to design mission specifications has shown potential to significantly reduce fuel burn for existing aircraft.\textsuperscript{2} Use of technological improvements like the introduction of composites in aircraft (like in the Boeing 787) has resulted in lighter, more efficient aircraft while not compromising structural integrity. Studies have been done on the effect of replacing turbofan engines with open rotors to improve the efficiency of propulsion systems thus reducing fuel burn,\textsuperscript{3, 4} use of laminar flow wings\textsuperscript{5} / nacelles\textsuperscript{6} and boundary layer ingesting propulsion systems.\textsuperscript{7} These are all technological improvements that promise a reduction in fuel burn while not resulting in massive changes to the aircraft configuration.

Studies\textsuperscript{8, 9} have shown that if we depart from the conventional tube-and-wing configurations and move towards unconventional configurations, fuel burn reductions that are much more significant compared to improvements obtained from individual technology changes, can be obtained. The Double Bubble configuration,\textsuperscript{9} designed by MIT and the blended wing body\textsuperscript{10} configuration have shown promise for a significant reduction in fuel burn. Truss and strut-braced wing configurations\textsuperscript{11} that permit much larger wing spans, resulting in much larger aspect ratios and much lower induced drag, are also being studied widely.

However, for conceptual design of these unconventional configurations, the typical correlation-based methods used for tube-and-wing aircraft do not work well. Accurate aerodynamic estimates are tricky to obtain using correlations (e.g., modeling the effect of the strut for the strut-braced wing). Structural weight estimation, which requires knowledge of the actual loads associated with these configurations, is also a challenging problem.\textsuperscript{12–18} Thus, physics-based methods and knowledge of aircraft geometry are required at the conceptual design stage itself in order to be able to accurately design such aircraft.

In this paper, we describe an aircraft design and optimization framework that uses SUAVE,\textsuperscript{19} a multifidelity aircraft conceptual design code for the mission analysis, coupled with a geometry generation tool,
GeoMACH, high fidelity aerodynamics using vortex lattice methods and CFD using SU2 and finite element based structural weight estimation. This framework is then used to perform conceptual design studies and MDO on a strut-braced wing configuration. Section II describes the design framework and design tools developed and used for this work. Section III then describes the application of this framework to the analysis and design/optimization of a strut-braced wing configuration. Finally Section IV summarizes the work done described in this paper.

II. Methodology/Design Framework

In this section, we describe the design methodology/framework developed as part of this work. The subsections elaborate on the conceptual design environment, the finite element-based weight estimation framework, geometry generation tool, high fidelity aerodynamics and how these are coupled to ensure an automated design process.

![Design framework diagram](https://example.com/design-framework.png)

**Figure 1. Design framework.**

In order to design unconventional aircraft configurations and to accurately model the aircraft performance, it is essential to include information regarding both the aircraft geometry and the physics in the performance analysis. The design framework developed is illustrated in Figure 1. SUAVE, a conceptual design framework under development at the Aerospace Design Lab at Stanford, has been used as the mission solver. A python interface is developed to link the aerodynamic, geometric generation and weight estimation tools developed/used in this work with SUAVE as shown in Figure 1. SUAVE handles the stability analysis, field length computation and mission performance prediction (like fuel burn). The generation and manipulation of the aircraft geometry is performed using GeoMACH, a geometry generation framework developed at the Multidisciplinary Design and Optimization group at the University of Michigan, Ann Arbor. For the aerodynamic prediction, employ either vortex lattice methods or Euler simulations using SU2 combined with handbook based drag prediction methods. Finite element-based weight estimates have been used to obtain an accurate estimate of the aircraft weight and to size its primary structure. Integrating these high fidelity models requires a conceptual design framework that supports the use of multiple fidelity levels and the ability to plug in new models without significant modification to the design tool. Next, we describe each of these tools in detail.

A. Stanford University Aerospace Vehicle Environment

SUAVE is a conceptual design tool capable of performing multi-disciplinary analysis at multiple levels of fidelity on an aircraft configuration and combining the results from the different disciplines to obtain per-
formance estimates for the aircraft over a simulated mission. It has a number of low and medium fidelity analysis capabilities for aerodynamics, structures and weight estimation, stability and propulsion analysis. The flexible framework also allows the user to couple external analysis modules in conjunction with the existing internal analysis modules. In this work, we link the external geometry, aerodynamics and weight estimation modules, and SUAVE provides an estimate of the aircraft performance over a specified mission. SUAVE’s optimization interface, built around pyOpt, \(^{23}\) VyPy \(^{24}\) allows us to perform optimization studies on the specified aircraft. For this study, we are interested in optimizing the strut-braced wing for fuel burn minimization.

B. Geometry and Mesh Generation

Geomach, \(^{20}\) is used for automated geometry and structural mesh generation. The aircraft is modeled as a combination of wing and fuselage components with junction elements, which permits smooth connections between the different components. Once the basic aircraft component intersections are set up, the different components are scaled based on the specified dimensions and locations of the components. GeoMACH allows the user to specify design variables that map to the geometric dimensions of the different components, which enables design based on parameters like root chord, tip chord, wing span that can easily be obtained from conceptual design environments. Once the parameters are passed in, an outer mould line for the aircraft is generated which is used to generate a computational fluid dynamics mesh or passed on to lower fidelity methods for load generation (described in Section D).

![Structural mesh of a strut braced wing and CRM aircraft](image)

(a) Structural mesh of a strut braced wing.  (b) Structural mesh of the CRM aircraft.

Figure 2. Geometry and structural mesh generation using GeoMACH

GeoMACH also allows the user to parameterize the internal structure of the aircraft. Thus for a wing, the number of internal ribs and spars can be specified. A finite element mesh of the aircraft can then be generated in an automated fashion, as shown in Figure 2, and this is used for FEA-based structural sizing (described in Section E).

C. Aerodynamics

Accurate estimation of aerodynamics is also critical to the aircraft design process. Results from the aerodynamic analysis are required for many different aspects of performance estimation. Accurate prediction of the aircraft lift coefficient \(C_L\) and drag coefficient \(C_D\) are essential to the prediction of fuel burn. Accuracy of the stability parameters, be it the stability margin or the moment coefficients, also depend heavily on the aerodynamic analysis.
1. **Low-fidelity aerodynamics**

Lift and drag estimates are obtained using the correlation-based methods set up in the conceptual design framework SUAVE. A vortex lattice method is used to compute the aircraft lift.

2. **High-fidelity aerodynamics**

Accurately predicting pressure distributions on the surface of a vehicle is essential for the generation of the aerodynamic loads that are necessary for structural sizing. A simple loading methodology using distribution functions is described in Section D.1. However, moving to unconventional configurations typically introduces geometric complexities that are not well handled by existing, simpler methods and often require computational fluid dynamics (CFD). In this scenario, the aerodynamics of complex geometries can be treated in a more accurate manner by solving the fluid equations in partial differential equation (PDE) form on unstructured computational grids. In particular, the Euler or Reynolds-averaged Navier-Stokes (RANS) equations will provide our high-fidelity predictions of the pressure and skin friction on the aircraft surfaces.

All high-fidelity aerodynamic calculations are carried out with the SU2 software suite. This collection of C++ codes is built specifically for PDE analysis and PDE-constrained optimization on unstructured meshes, and it is particularly well suited for aerodynamic shape design with complex geometries. Modules for performing flow and adjoint solutions, acquiring gradient information by projecting surface sensitivities into the design space, and mesh deformation techniques are included in the suite, among others.

The suite components are integrated within a Python framework, which allows for the automation of more complex tasks that involve multiple modules, such as optimal shape design or multi-physics problems. In this work, the existing Python framework for SU2 has been extended to increase interoperability with external tools and libraries, improve the computational performance and scalability of the overall framework, and to add flexibility for expanding it to new multi-disciplinary problems or multi-component tasks unforeseen by the authors. To accomplish this, the SU2 suite was wrapped for Python with a new interface layer using SWIG, which ensures that all of the classes and functions implemented in SU2 can be accessed from Python and that calls any data transfer occurs directly through memory (rather than file I/O). Furthermore, the same distributed memory computing model in SU2 with the Message Passing Interface (MPI) is maintained. The result is an SU2 package that can be imported in Python and tightly-coupled to other analysis packages through driver scripts.

D. **Load generation**

A variety of loads need to be applied to the aircraft to ensure that the structural elements are appropriately sized. In this section we describe the different structural sizing loads applied to the aircraft configuration.

1. **Flight loads**

The 2.5g maneuver loading condition is a critical loading condition that is used in the cases studied here. The -1g loading condition is another important condition that is used to size the structure.

The aerodynamic loads on the aircraft surface are generated using spanwise and chordwise lift distribution functions (elliptical or triangular spanwise and rectangular or triangular chordwise). For some cases, CFD-based loading is also used, as described in Section C.2, with these CFD loads transferred to the aircraft structural mesh using a consistent and conservative load transfer mechanism.

2. **Internal Pressure loads**

Modeling the cabin pressure loads is important for structural sizing as it determines the thickness of the fuselage skin/stringers. For this study, a 1.5P cabin loading is used, where P is the atmospheric pressure at 8000 ft (10.9 psi), because this is the standard cabin pressure maintained in flight.

3. **Fuel loads / Payload**

The fuel load is uniformly added to the lower wing surface. Similarly, the load exerted by the payload is uniformly distributed on the lower surface of the fuselage.
E. Structural sizing, Finite Element based Structural Weight Estimation

For unconventional configurations like the strut-braced wing, using correlation-based methods for aircraft weight estimation results in an inaccurate weight estimate. This is because most correlations are based on regressed aircraft data and an assumed material and loading. For unconventional configurations like the blended wing body and the strut-braced wing, we must consider the geometry, the aerodynamic loads that result due to the complex geometry, as well as the material used to model the structure for accurate weight estimation. Ensuring that the aircraft structure can withstand the different complex loading conditions is the only way to ascertain the minimum possible aircraft structural weight for a feasible design. Thus, a finite element based structural weight estimation framework is developed and used in this work. This framework, shown in Figure 3, uses a coarse model of the aircraft structure to perform a computationally inexpensive finite element analysis under different loading conditions and estimates the minimum thickness of the different structural members (and thus minimum structural weight) required to withstand the loads.

The module obtains the structural finite element mesh from GeoMACH and the aircraft sizing loads from the load generation module and generates the necessary files required for the finite element solver and optimizer. For this study, two structural optimization frameworks have been tried out, Nastran’s Solution 200 optimization capability and the Toolkit for Analysis of Composite Structures (TACS), which is a finite element based structural solver coupled with SNOPT through pyOpt. Both of these structural sizing frameworks are validated design tools, and the results obtained from both tools for this study are similar.

The structural design problem solved is the minimization of the weight of the structure while meeting the stress constraints by changing the thicknesses of the shell elements that form the aircraft structure as shown in Equation 1. Varying the thickness of every mesh element results in thousands of design variables, making the problem extremely expensive to solve. Therefore, the finite elements are grouped together into smaller sets with all the elements in a group having the same structural properties and element thicknesses. This results in a design problem with tens or hundreds of design variables which is a more computationally tractable design problem.

\[
\begin{align*}
\min_{x \in \mathbb{R}^N} & \text{ (weight of structure (kg))(x))} \\
\text{such that} & \quad g_i(x) \leq 0, \quad i = 1, \ldots, M,
\end{align*}
\]

where \(x\) is a vector of the design variables used for this study which in this case is the thickness of the shell elements and \(g_i\) are the constraints enforced to ensure feasibility of the design, in this case the Von-Mises stress of the FEM elements. The weight estimate and the stresses computed are passed on to the conceptual design environment driving the design / optimization run.

![Figure 3. Weight estimation framework](image-url)
III. Results

After describing the design tools developed and used in this paper, we now move on to the results we have obtained during this work. Section III.A describes the validation of the weight estimation framework using a CRM wing configuration and the effect of mesh refinement. In section III.B, we describe the weight estimates obtained from a strut-braced wing configuration. Then in subsection III.C, we describe the redesign/optimization of this configuration for the minimization of fuel burn.

A. CRM wing analysis

1. Validation

The selected validation case shown here is the structural weight estimation of the undeformed NASA CRM wing configuration (called the $\mu$CRM configuration). Aerostructural design and optimization has been performed on this configuration in the work by Kenway, Martins and Kennedy. An outer mould line of the geometry and a finite element structural mesh were obtained from the authors in order to perform a one-to-one validation. The structural mesh uses shell elements to model the skin and ribs of the structure. As described in Section C.2, an Euler simulation is run on this configuration using SU2. The simulation is configured to obtain a 2.5g maneuver loading case at Mach 0.65 at 10000 ft.

The problem uses 260 design variables for the shell element thicknesses. The Von-Mises stress at each element is constrained to be below the yield stress of the material, which in this case is a 7000 series aluminium alloy with a Young’s modulus of 70 GPa, a poisson ratio of 0.33 and a density of 2780 kg/m$^3$. The Figure 5 shows the convergence history of the objective function during this optimization. The thicknesses of different elements in the structure are shown in Figure 4(a).

The wing weight is estimated to be 12410 kg, which is in close agreement with the weight obtained by Kenway, Martins and Kennedy (12263 kg). Thus, the finite element-based framework provides accurate weight estimates.

2. Effect of mesh refinement

Next we look at the CRM wing geometry generated using GeoMACH (Figure 5(a)) and use the loads generated using the low-fidelity loading methodology. For the GeoMACH generated meshes, the effect of mesh refinement on the weight estimate needs to be considered. The size of the structural elements in the finite element mesh affects the weight estimate. Finer structural meshes permit better prediction of the stresses (higher stress values are obtained). Thus as the meshes are refined (uniform refinement here) the
weight estimates become more accurate (increase) as shown in Figure 6. However as the mesh is refined, the computational cost of each finite element evaluation (direct and adjoint) increases making the structural optimizations significantly more computationally expensive. So for this study we compromise between mesh refinement and computational cost. For the CRM wing, we see (Figure 6) that beyond 15000 elements, the prediction of the structural weight is within 5% of the results of the finest mesh (around 45000 elements). This is deemed acceptable for conceptual design. So for the remaining studies (strut-braced wing) the average element size is chosen to be the same as the mesh with around 15000 elements. The optimal thicknesses for the CRM configuration (251 design variables) is shown in Figure 5(b).

![Figure 5. Effect of mesh refinement on the CRM wing weight estimation prediction.](image)

(b) Optimal thickness distribution for the CRM wing.

(a) CRM wing structure generated by GeoMACH.

B. Strut-braced aircraft analysis

1. Baseline Configuration

For the strut-braced wing, the baseline geometry modeled in GeoMACH is based on the N+4 Truss-braced wing geometry from the NASA Subsonic ultra green aircraft research phase II project.\textsuperscript{11} The baseline geometric parameters of the aircraft are shown in Table 1 and the configuration is shown in Figure 8. The aircraft is intended as a next-gen replacement to the conventional Boeing 737-800. In order to reduce the computational cost of structural optimization, only the main-wing, strut and the fuselage are modeled for FEA-based weight estimation. The weight of the t-tail is obtained from handbook methods.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Strut-Braced</th>
<th>Conventional</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wing Span (ft)</td>
<td>170</td>
<td>117</td>
</tr>
<tr>
<td>Wing sweep (deg)</td>
<td>12.5</td>
<td>24</td>
</tr>
<tr>
<td>Wing area (ft\textsuperscript{2})</td>
<td>1477</td>
<td>1440</td>
</tr>
<tr>
<td>MTOW (lbs)</td>
<td>156000</td>
<td>174200</td>
</tr>
</tbody>
</table>

For this problem, we use the same material and the same constraints as for the $\mu$CRM described in section III.A.1. The loads are generated using the low fidelity load generation method described in Section II.D.
2. Choice of structural design variables

The selection of the design variables of the structural optimization is critical to the weight estimation process. The wing, fuselage and the strut are broken down into sections, the wing and the strut in the spanwise directions and the fuselage in the direction of the freestream. The wing and the strut sections are further broken down into the upper and lower skin and spars and ribs. The fuselage contains skin and internal structure elements. The structural elements that fall in each of these sub-sections are chosen to have the same element thicknesses. Thus as the number of sections increase, the number of design variables increase. For the case with one section (Figure 8(a)), there are 31 structural design variables comprising one upper and one lower wing skin, wing tip, spar and rib thickness for the main wing, one upper and one lower wing skin and one spar and rib for the strut and 4 fuselage skins (upper, lower, left, right), a root and a tip element, and one set of transverse and longitudinal elements (each set of fuselage internal elements is broken into 4 subsets) for the fuselage internal structure along with thicknesses for the intersection components. As the number of sections increases, the number of design variables (thicknesses) for wing upper and lower skins and spars and ribs as well as the fuselage skin and transverse and longitudinal internal structure elements each are scaled by the number of sections.

The choice of the number of design variables to be used for each component affects the weight prediction. As the number of design variables per component is increased, the structural weight of the component decreases. Localized increase in stresses can be countered by locally increasing the element thickness, as shown in Figure 8. However, increasing number of design variables results in increased computational cost as well. We see from Figure 7 that beyond 14 sections (for each component) the weights do not change significantly with increasing number of design variables/sections. Thus for the aircraft optimizations described in Section III.C, we restrict ourselves to 14 sections each for the wing, strut and fuselage resulting in a total of 291 design variables. A smarter choice of the design variables, like clustering a few design variables near the wing/fuselage intersection and leaving the rest of the fuselage element thicknesses as 1 design variable could result in a reduced number of design variables with the same accuracy. However these have not been considered here.

![Figure 6. Effect of mesh refinement on the weight estimate](image)
C. Design and optimization of the strut-braced wing aircraft

1. Optimization Problem

Next, we consider the optimization of the strut-braced wing configuration for optimal fuel burn over a design mission. For this study a cruise mission of 2950 nautical miles is chosen as the design mission.

The optimization problem is formulated in Equation 2 i.e.

\[
\min_{x \in \mathbb{R}^N} \text{Kg fuel}(x)
\]

such that \( g_i(x) \leq 0, i = 1, \ldots, M, \) \hspace{1cm} (2)

where \( x \) is a vector of the design variables (shown in Table 2) used for this study and \( g_i \) are the constraints (shown in Table 3) enforced to ensure feasibility of the design.

In order to demonstrate the design framework’s optimization capabilities, a simple optimization problem is chosen with the maximum takeoff gross weight (MTOW), design thrust, main wing taper ratio and the cruise altitude chosen as the design variables. The aircraft is also required to meet the takeoff (TOFL) and landing field length (LFL) constraints and the second segment climb gradient with engine out constraint. The max throttle of the engine model is constrained to be less than 1.0 and the zero fuel margin (which is the difference between the landing weight of the aircraft and the sum of the operating empty weight, payload and reserve fuel) is constrained to be positive. Thus the optimization problem is more of a sizing problem. As each function evaluation of the aircraft level optimization involves a structural optimization (for weight estimation), the aircraft optimization process becomes computationally expensive (for conceptual design). Gaussian Process Reduction (GPR) is used to build a surrogate of the structural sizing/weight estimation process.
(a) Design with 1 section (each component), 31 design variables.

(b) Design with 4 sections (each component), 91 design variables.

(c) Design with 14 sections (each component), 291 design variables.

(d) Design with 24 sections (each component), 491 design variables.

Figure 8. The effect of increasing design variables on a strut-braced wing configuration
2. GPR based reduction of structural optimization

In order to reduce the cost associated with the structural sizing during aircraft level optimizations, a surrogate of primary structural weight is constructed using Gaussian Process Reduction. The GPR capabilities\textsuperscript{31} in VyPy are leveraged for this purpose. In the optimization problem described in section III.C.1 only the MTOW and the wing taper affect of the primary structural weight. The design space is sampled at 50 locations using a latin hypercube sampling methodology for wing taper and MTOW and the primary structural weight at that location is obtained. GPR is used on this data to build a surrogate for the primary structural weight. This surrogate is then used in the optimization loop. The cost of the aircraft level optimizations are reduced by multiple orders of magnitude. For more complex optimization problems with a large number of aircraft level design variables (like wing span, root and section chords), the design parameters that affect the primary structural weight can be obtained using principal component analysis or using active subspaces on the full problem and then an GPR based surrogate can be built using those design variables that affect the primary structural weight.

3. Optimization Results

Once the GPR is included in the aircraft level optimization loop, the optimizations are performed. The initial values for the aircraft as shown in Table 2 are similar to the B737-800. The aircraft is not feasible as shown by the initial constraints (table 3) with both the takeoff and landing field lengths not met. The fuel burn for the initial design is 32700 lbs.

The optimizer is able to reduce the MTOW to 157000 lbs (compared to 156000 lbs for strut-braced configuration with LNG based gas turbine in the NASA Ultra Green Aircraft Research Phase II report\textsuperscript{11}). The cruise altitude is increased from 35000 ft to 41000 ft. The design thrust is increased to meet the field length constraints. The cruise thrust is much lower though as the maximum throttle used (not including takeoff and landing) is 0.71 (1.0 implies full throttle). The aircraft meets all the constraints and the fuel burn is reduced to 28900 lbs. While more design variables (like wing span, chords, and wing location, strut dimensions and location) and constraints are required to obtain a realistic design, the optimization demonstrates the ability of the framework to handle design optimizations of unconventional aircraft configurations in an automated fashion.

<table>
<thead>
<tr>
<th>Design Variables</th>
<th>lower bound</th>
<th>initial</th>
<th>final</th>
<th>upper bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>MTOW (lbs)</td>
<td>44092</td>
<td>174200</td>
<td>157000</td>
<td>264554</td>
</tr>
<tr>
<td>Design Thrust (lbf)</td>
<td>2248</td>
<td>7874</td>
<td>11313</td>
<td>22480</td>
</tr>
<tr>
<td>wing taper ratio</td>
<td>0.25</td>
<td>0.35</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>Cruise Altitude (ft)</td>
<td>30000</td>
<td>35000</td>
<td>41000</td>
<td>45000</td>
</tr>
</tbody>
</table>

Table 2. List of the aircraft design variables with bounds for the strut-braced wing case.

<table>
<thead>
<tr>
<th>Constraints</th>
<th>Optimum</th>
<th>Initial</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zero Fuel Margin(kg) &gt; 0</td>
<td>1e-5</td>
<td>10000</td>
</tr>
<tr>
<td>TOFL(ft) &lt; 7875</td>
<td>7874</td>
<td>15000</td>
</tr>
<tr>
<td>LFL(ft) &lt; 5570</td>
<td>5570</td>
<td>7316</td>
</tr>
<tr>
<td>Second Segment Climb Gradient &gt; 0.024</td>
<td>0.1</td>
<td>0.05</td>
</tr>
<tr>
<td>Max throttle &lt; 1.0</td>
<td>0.71</td>
<td>0.85</td>
</tr>
</tbody>
</table>

Table 3. List of the aircraft design constraints with bounds for the strut-braced wing case.

IV. Conclusions

In this paper we have presented a multi-fidelity design framework that is capable of performing conceptual analysis, design and optimization on unconventional aircraft configurations using physics-based meth-
V. Acknowledgements

The authors would like to thank the SUAVE developers team and the SU2 developers team for providing them with codes they could use as a foundation. They would like to thank Dr. John Hwang and Dr. Joaquim Martins for their use of GeoMACH which has also proved critical for this study. The authors gratefully appreciate Dr. Joaquim Martins, Dr. Graeme Kennedy and Dr. Gaetan Kenway for providing them with TACS and the µCRM geometry/structural mesh and validation data. Anil Variyar would like to acknowledge the Stanford Graduate Fellowship for funding him for the academic years 2013-15 at Stanford.

References


